# I. *The Construction of a Quadratrix to the Circle, being the Curv described by its Equable Evolution.*

1. BY the *Equable Evolution* of a Circle, I mean such a gradual approach of its Periferie to *Rectitude*, as that all its parts do *together*, and *equally* evolve or unbend: or so that the same Line becomes successively a less and less Arc of a reciprocally greater Circle.

2. Let A H K A (Fig. 6.) be the Periferie of a Circle. A E a Tangent to the point A. Let this Circular Line be suppos'd cut or divided at A, and then to unbend (like a *Spring*) its upper end remaining fixt to its Tangent A E, whilst the other parts do *Equally Evolve* or extend themselves thorough all the degrees of less Curvature (as in ABD, AMC, *&c.*) till they become straight in coincidence with the Tangent AE.

3. Let AMC be the *Evolving Curv* in any middle position between its first and last. Joyn the fixt end A, and the moving end C, by the Chord-line AC, intersecting the first Circle at H. I say that AMC is a *like* Segment to A n H, cut off in the first Circle by the Chord A H. For, by the supposition of AMC is the Arc of a Circle, having AE a Tangent common both to it and A n H, and both Arcs are terminated in the same Right line AC.

4. Hence the Curv ADCE (describ'd by the moving end of the Periferie in its Evolution) may be thus constructed. Let the Circle AHKA be by bisections divided into any number of equal parts. Let H be one of the points of such division. Then say, as the number of equal parts in the Arc A n A: is to the number of parts in the whole Periferie A H K A :: so is the Chord A H: to a fourth Line, which let be A C in A H produc'd. So is C a point in the Curve A D C E.

5. Dem. Upon A C describe A M C, an Arc *like* to the Arc A n H. Whence— A H: A C :: A n H: A M C. But by construction, A H : A C :: A n H : perif: A H K A, therefore is the Arc AMC equal to the whole Periferie AHKA and

and like to the Arc A n H. Consequently AMC represents the Evolving Periferie, in a position like to the Arc A n H, and C is the describing point.

6. After the same manner may be found other points thro which the Curv may be drawn. But here (as in the old *Quadratrix* of *Dinostratus*) the point E cannot be precisely determined but the Curv may be brought so near it, that its flexure or tendency will so lead to the point E, that A E shall be near enough to the truth for common uses.

7. Supposing the point E found, a *Tangent* to any point of the Curv may be drawn: and supposing a *Tangent* drawn, the point E may be determined; the property of the Tangent being this, that supposing R T a Tangent to the point C and C A, C E, drawn from C to each end of the rectify'd Circle, the Angle ACT (the lesser angle that AC makes with the tangent) is equal to the tangent made by the 2 Lines drawn from C.

8. c be a point in the Quadratrix indifinitely near to C; and draw A c intersecting AHKA in h, and AMC in o. To A c as a chord, draw the Arc Amc like unto the Arc A n a. To the point c of the Arc A M C draw the Tangent CL = A E, and joyn L A: So is oC an indefinitely little particle of the Arc coincident with its Tangent.

9. Because of the like Segments AnhA, AMoA, AmcA, as chord Ac: to chord lo :: So is Arc Amc (= AMC): to Arc AMo. Or Ac : Ao :: Amc (= AMC): AMo. And dividing Ac - Ao (= co) : Ao :: AMc—AMo (=Co): AMo. That is, co : Ao :: Co : AMo. and alternately, co : Co :: Ao : AMo. Put AC for Ao, and AMC for AMo (as differing infinitely little) and then 'tis co : Co :: AE : AMC. But by construction CL=AE =AMC whence co : Co :: AC : CL and the Angle LCA=Coc. (oc being infinitely near to AC, is therefore parallel to it.) and therefore Coc, ACL are *like Triangles*.

10. Because of CL =AE, Ang. EAC = LCA. (CL and EA being Tangents to the two ends of the same circular Arch AMC, make equal Angles with its chord AC.) and AC common to both, the Triangles EAC, ACL are *like* and *equal*: therefore are all three Coc, ACL, EAC *like Triangles*. Whence it follows, that the Angle ACE (in the Triangle EAC) is equal to the Angle ocC (in the Triangle coC.) But ocC = ACT because oc and AC are parallel; therefore Ang. ACE =ACT. Q.E.D.